Resource Allocation in LTE-Advanced Network: A Collaborative Market Game Approach

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Abstract: **The essence of high Quality of Service (QoS) provisioning in any network is centred on optimization which can be approached from different perspectives depending on the network operator's target and Key Performance Indicator (KPI) parameter tweaking ability. In this article, we present a collaborative resource allocation technique using the Market Game that utilizes the Shapley value solution concept which is a method used in the field of Political-Economy for fair distribution of resources as exemplified in a welfare state. Modelled on the Long Term Evolution (LTE) Advanced Network, the results show an improvement in the throughputs achieved by users of the network.**

Keywords-LTE-Advanced; Resource Allocation; Collaboration; Fairness; Optimization

I. INTRODUCTION

In recent years, there has been a major leap in data communications. This is significantly demonstrated in the evolution of cellular radio networks over the past three decades. A period that has witnessed different phases of evolution: the first generations, second generations, third generation, fourth generation and now the fifth generation of network. This is also evidenced in the emergence of smarter communication devices that give us the ability to communicate ubiquitously. This change comes with great demand on the system especially with respect to capacity requirement to enable larger data of different classes be transmitted [1]. As smarter devices keep emerging along with more applications with higher bandwidth requirement, the system has to keep up in providing guaranteed high Quality of Services (QoS).

The 3GPP adopted the Long Term Evolution (LTE) which over time evolved into the LTE-Advanced as the 4G standard that is envisaged to provide users with data rates as much as 1000Mbps on the downlink and 500Mbps on the uplink directions respectively [1].

In this article, we make an in-depth review of Resource Allocation (RA) for LTE-Advanced Networks and propose a Collaborative RA technique based on Market Game in order to perform efficient RA during network optimization. This paper is organized as follows: section II describes RA in Orthogonal Frequency Division Multiple Access (OFDMA) wireless network in the context of the LTE-Advanced; section III presents the related works. Section IV presents the proposed algorithm and V is a performance evaluation of the algorithm. VI concludes this article.

II. RESOURCE ALLOCATION

There are variants of RA techniques and scenarios in OFDMA systems both in the uplink and downlink directions. These schemes include centralized and distributed, instantaneous and ergodic, optimal and suboptimal, single cell and multicell, cooperative and non-cooperative techniques, to mention a few [5]. These schemes have been thoroughly investigated in the literature, for fairness and performance maximization mostly employing the classical Shannon capacity formulation [5] to present the models for expected rates which are tweaked to achieve desired Quality of Service (QoS) with power allocation as a major constraint. A major limitation as pointed out by [5] and supported by many authors is the fact that these techniques all depends on the Channel State Information (CSI) in order to allocate resources to a User Equipment (UE) which advertently results in delays and high resource consumption as the buffer queue gets longer whilst the data piles on the queue. Due to the scarcity of spectrum as well as the amount required to be invested before operators get good number of spectrum to operate, RA becomes a great focus to maximize performance, QoS and profit. The essence here is to maximize utilities such as the sum-rate and power for various UEs who are at every point in great need of data transmission.

In 4G (LTE-Advanced), OFDMA RA is basically performed on subcarrier and power allocation basis [5, 6] which is based on the user's rate requirements for services provided and their QoS requirements. The problem of RA in LTE-

Advanced Network has unarguably been investigated in greater depth. However, no other work in the literature, as would be seen in section III, has treated the problem as we presented using the Market Game Theory for better RA as well as guaranteed QoS provisioning. Our algorithm is a variant of the cooperative allocation technique that enables high rate for users without causing much delay as it used to be.

RA in LTE-Advanced is performed by the evolved Node B (eNB) where the OFDMA radio resources are allocated to users as Physical Resource Blocks (PRB) [7] which are the basic units of transmission in the OFDMA based LTE-Advanced network. There are different types of algorithm available for RA [7]: Proportional Fair (PF), Exponential PF (EXP-PF), Round Robin (RR), Max-Min Fair, Exponential Rule (EX-Rule), Max-Weight and Maximum Largest Weighted Delay First (M-LWDF) [8-12]. Using utility-based technique for an adaptive radio technique, [13] posited that there is a trade-off between fairness and efficiency in radio RA so that QoS can be satisfied.

Amongst these techniques, only EX-Rule and Max-Weight techniques are known to support both real time and non-real time data flows. Their objective functions maximize the system throughput (data rate) depending on the users' queue length. Therefore, a user may continue to transmit until its queue is comparatively reduced to the detriment of the others. In our algorithms therefore, we seek to improve the system throughput considering both real time and non-real time data flows, not minding the users' queues. In which case, we look at the required SINR of the users and constrain them through the eNB to collaborate with one another in order to ensure that all users, especially those with lower rate; either longer/shorter queues are able to get good QoS satisfaction for their service classes and also ensure that fairness is observed in the distribution of available resources.

III. RELATED WORKS

To address the problem of managing dynamic network conditions such as unreliability in network links that result in frequent service outages, [14] proposed a collaborative architecture which takes advantage of the availability of multiple network nodes from different administrative domains in order to maximize connectivity and high service availability. There have been the ideas of virtualization through running multiple applications, concurrent application provisioning and programming abstractions. The work in [14] adopts fixed channel allocation in order to minimize interference in the system. The technique in [14] is based on an event triggered by a node or on demand triggered by gateways. Updates are sent between nodes through wired interconnection in order to exchange information that would enable cooperation.

Using non-cooperative game, [15] devised a semi distributed algorithm in LTE-Advanced enabled Femto network that considered the uplink RA in order to alleviate the macro-femto interference and maximize the femto cell capacity. The technique charges femto users a price proportional to the amount of interference which it causes to the macrocell. It employs the modified iterative water filling based power allocation algorithm in which sub-channels are initially allocated to users then power is allocated to each sub-channel. The algorithm proposed in [15] performs interference aware femto-cell uplink RA which helps to improve the system capacity.

By proving the efficiency of the "*binary power allocation*" scheme, [16] proposed a joint power allocation scheme based on the binary power allocation technique which proved to be also efficient in performing RA and providing higher system capacity. The theory employed the Lagrangian multiplier which performs the equation relaxation. The scheme also, set in a femto network, is non-cooperative in nature but quite efficient on implementation.

To solve the problem of abrupt network fault through self-healing as contained in Self Organizing Networks (SON), [17] proposed a collaborative RA algorithm which aims to rescue users in disabled femtos with minimal effect on the system capacity. They designed a centralized algorithm based on water filing (modified iterative water filling and improved iterative water filling) to solve single user power control problems in multicell multiuser control power problems therefore, helping users who cannot be served in a faulty cell. In this algorithm, power is equally allocated on all sub channels. The authors in [18] used round robin technique to perform the RA in cooperative relay network. In [19], the problem of channel allocation was solved through collaborative coalition formation in a femto enabled system in which users form a collaborative coalition to increase the femto-cell's total throughput and later demand a payoff proportional to their achievable throughput in the coalition.

The work in [20] proposed a game theoretical framework based on the Nash bargaining of the cooperative game theory to provide solutions for the ideologies of cooperation where either a price is paid or a penalty is scored to enable cooperation. Cooperative communications requires careful RA and coordination techniques in order to realize and maximize the gains of employing the techniques. Also, the authors in [21] solved this by deriving theoretical expressions focused on asymmetric radio RA.

To enable cooperation in wireless networks, [22] used the idea of coalition games to construct a simple distributed merge-and-split algorithm which maximizes their rate utilities with power as the constraints where antennae are accounted for. The work basically provides the rules and principles of cooperation/collaboration in wireless networks. As initially stated, a lot of work have been done on game theory and collaboration such as [23] but these works however have been tailored towards cognitive radio networks which leaves LTE-Advanced Networks unattended. We therefore in this article have considered what others have in a bit left out of consideration to devise an effective RA through a Collaborative Algorithm in the LTE-Advanced Relay Network.

IV. COLLABORATIVE MARKET GAME

Market games are simple models of exchange economy that focus on participant exchange and effect on the market structure. It gives rise to competitive outcomes when agents lack strong market power [24]. A major constraints here is that consumers cannot bid more money than they received from the sale of the commodity [25]. This Game Model is a member of the Class of Cooperative Games with Transferable Utility [25, 26]. It is also an Optimization technique used in the fields of Economics and Political Economies to model the optimization process of exchange economies [27, 28] as well as social behaviour to create a balanced welfare system for individuals who may not live up to required standards. It ensures that participants are properly served as much as the limits of the available resources may permit [29]. In this article, we assume that coalition is formed as a process of collaboration in order to enable UEs improve their rates. As the results would show, we also measured the stability of the coalition where our algorithm proved to be effective in this respect. The algorithm assumes the Shapley Value solution concept [32] in order to ensure that there is fairness in the distribution of allocation.

V. SYSTEM MODEL

This algorithm is assumed to be totally under the control of the evolved NodeB (eNB) since users are not aware of each other and their requirements but would be constrained to collaborate in order to achieve their service's requirements especially at the cell edge. We consider the downlink transmission of the network as the core of our problem. Previous literatures have shown that a UE's throughput/data rate is a function of the achievable SINR. Using the equation (1) [30], for the ith UE, we define the SINR as:

$$\gamma_i = \frac{P_s K_1 10^{\frac{\sigma_{s1}}{10}\xi}}{d^{\alpha_1}\left(\sum P_j + N_{UE}\right)} \tag*{1)}$$

Where $\gamma_i = SINR$

$P_s =$ Transmission power of the eNB

$P_j =$ Interference power from the eNB$_j$

$d =$ Disatnce between UE and eNB

K_1 and α are constant parameters deduced from propagation loss

$K_1 = 10^{-14.178} * 10^{-\frac{L_{Trans}}{10}}$

$L_{trans} =$ Penetration loss

$\alpha_1 = 3.5$

$N_{UE} =$ UE thermal noise

$\xi =$ Standard normal random variable that models fading

$\sigma_s =$ Standard deviation of shadow fading

The UE's data rate/throughput is defined as

$$C_i = \frac{B}{N} * \log_2(1 + \gamma_i) \tag{2}$$

C_i is the throughput of the ith user in the network. B is the system bandwidth and N is the number of allocated subchannels. At every Transmission Time Interval (TTI), there are K users occupying the network who would be needing the available resources. This implies that the available radio resources would need to be shared amongst these users, depending of course, on their demands. A user would therefore bid u portion of the available cell capacity, C_T at every t; that is C_T. During t, in a cell with K users, the ith user would therefore demand u_i which would receive a service rate of [31]:

$$v_i = \left(\frac{u_i}{\sum_{i=1}^{K} u_k}\right) * C_T \tag{3}$$

From (3), u_i is the ith user's demand for which the rate v_i is required to effectively satisfy it's service. Therefore, combining (2) and (3);

$$v_i = \left(\frac{C_i}{\sum_{i=1}^{K} C_k}\right) * C_T \tag{4}$$

Where C_k is the sum total of demand for all users in the coalition. The price to be paid per unit resource for this demand is given as:

$$\frac{\left(\sum_{i=1}^{K} C_k\right)}{C_T} \tag{5}$$

$$\begin{aligned} C_T \\ &= numberofsector * spectralefficiency \\ &* bandwidth \\ &* busyhourloading \end{aligned} \tag{6}$$

The price is an indicator of resource demands as well as a feedback for congestion. It is in effect the the UE's required SINR, γ'. For a ith user to transmit, it is required that $\gamma_i \geq \gamma_i'$.

$$\gamma_i' = \frac{P_s * G_{T_x} * G_{R_x}}{\zeta_{T_x} * \zeta_{R_x} * G_{R_x} * l_p} \tag{7}$$

Where G_{T_x} is the transmitter gains, G_{R_x} is the receiver gains, ζ_{T_x} is the transmitter losses, ζ_{R_x} is the receiver losses. l_p is the pathloss. Using the COST 231-Wlfisch-Ikegami pathloss model and considering the line of sight (LOS) model;

$$l_p = 42.6 + 26 * \log\left(\frac{d}{Km}\right) + 20 * \log\left(\frac{f}{MHz}\right) \quad (8)$$

$d =$ distance between the ith user and the eNB

$f =$ network operating frequency (800 to 2000MHz)

A strategic game involving coalition formation is defined as $(N, v); S \subseteq N$ for which a payoff v is defined as $v: \mathbb{R}^+ \longrightarrow 2^N$. [32-34] S is a coalition formed from among the grand coalition, N which is the number of players, henceforth referred to as users. v is a utility function defined on S to distribute the payoff to users as a result of being in the coalition. For this algorithm, we assume that the coalition is formed spontaneously. Given n players in the game, the set of possible coalition formed in the game 2^N has 2^n elements.

With fairness in mind, in order to ensure that the cell end users are satisfied, we look to the Shapley value solution concept [34-37] so that a fair distribution of the earned payoff occurs in the process of allocation. The Shapley Value, $\emptyset_i(N, v)$ earned by the ith user as its allocation for transmission, from (4) is given as:

$$\emptyset_i(N, v) = \frac{1}{N} \sum_{S \subseteq N\{i\}} |S|!\,(|N| - |S| - 1)! \left[\left(\left(\frac{C_i}{\sum_{i=1}^{K} C_k} \right) * C_T \right) - \sum_{i=1}^{K} v_k \right] \quad (9)$$

v_k is the total payoff of all members of the coalition and the marginal contribution of the ith user in this coalition is given as $\left[\left(\left(\frac{C_i}{\sum_{i=1}^{K} C_k} \right) * C_T \right) - \sum_{i=1}^{K} v_k \right]$. This means that $\forall i \in S \subseteq N$; $\left(\left(\frac{C_i}{\sum_{i=1}^{K} C_k} \right) * C_T \right) \geq \sum_{i=1}^{K} v_k$.

The fairness of payoff requires that the values obeys some axioms:

i. Symmetry: for two users i and j, say at the cell edge whose contributions in the coalition are the same, their payoff have to be the same:

$$\begin{cases} \left(\frac{C_i}{\sum_{i=1}^{K} C_k} \right) * C_T = \left(\frac{C_j}{\sum_{i=1}^{K} C_k} \right) * C_T \\ \emptyset_i(N, v) = \emptyset_j(N, v) \end{cases} \quad (10)$$

ii. Dummy Player: the ith player of the coalition is a dummy player if its contribution amounts to zero:

$$\forall S: \frac{C_i}{\sum_{i=1}^{K} C_k} = \sum_{i=1}^{K} v_k \quad (11)$$

(10) translates that $\forall v$, if i is a dummy player, then $\emptyset_i(N, v) = 0$.

iii. Additivity: this axiom supposes that if the game is divided into two parts, then the payments, $v = v_1 + v_2$. It therefore means that,

$$\emptyset_i(N, v_1 + v_2) = \emptyset_i(N, v_1) + \emptyset_i(N, v_2) \quad (12)$$

$\therefore \quad (N, v_1 + v_2) = \big(v_1(t) + v_2(t)\big)(S) = v_1(t)(S) + v_2(t)(S);\ \forall S \subseteq N.$

The Shapley value is extremely difficult seeing, intuitively that the coalition ordering involves a prohibitively large amount of calculations. We therefore employ the Owen multi-linear extension method [33] to speed up the computation thus reducing the computational complexity and delay in packet scheduling.

According to the Owen Method [33], there exists a unique multilinear function, $\overline{v}: [0,1]^n \rightarrow$ ℝn that coincides with 2^N. i.e., $\overline{v}(x_1, \dots, x_n) = \sum_{S \subseteq N \setminus \{i\}} (\prod_{i \in S} x_i \prod_{i \notin S} (1 - x_i)) v(S)$ $\forall (x_1, \dots, x_n) \in [1,0]^n$. x_i $i = 1 \dots n$ is the payoff for individual players. $\prod_{i \in S} x_i \prod_{i \notin S} (1 - x_i) = P_x(\chi_S)$ is a product probability defined on $\{0,1\}^N$ $\forall S \subseteq N$. $P_x(\chi_S)$ is the probability of the formation of random coalition. Therefore, the expected value is given as: $\overline{v}(x_i) = \sum_{S \subseteq N} P_x(\chi_S) v(S) = E_{P_x}(v)$. From this, the Shapley value $(N, v); S \subseteq N$ with the multilinear extension $\overline{v}, \forall i \in N$, is given as [33]:

$$\emptyset_i(N, v) = \int_0^1 \frac{\partial \overline{v}}{\partial x_i}(t, \dots, t)\,dt \quad (12)$$

Carrier Frequency	2Ghz
System Bandwidth	20MHz
Spectra Efficiency	0.65
SINR Efficiency	0.95
Penetration Loss	10dBm
Outdoor Shadowing	8dB standard deviation
eNb Tx Power	46dBm
eNB Tx Gain	14dBi
UE Noise Figure	9dB
UE Gains	0
UE Tx Power	≈ -30dBm – 23dBm

Table 1: System Parameters

VI. PERFORMANCE EVALUATION

We set up a simulation in Matlab® using the parameters in Table 1 to evaluate our proposed algorithm. As mentioned earlier, the essence of the algorithm is to ensure that users at the cell edges are properly served as well as those in network blackhole areas. This is ensured by evaluating the marginal contributions of the users as presented in the system models. The results show users' behaviour when they are collaborating with others as well as when they are acting alone as evidenced in the other algorithms. Fig 1 presents the throughput of the users in the different scenarios of the model.

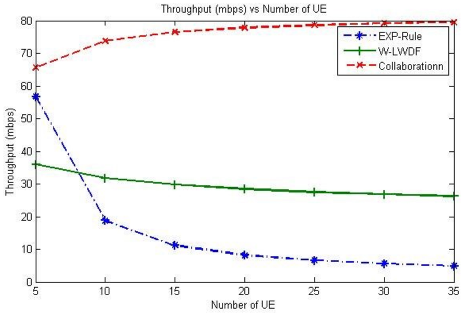

Fig 1: Throughput vs Number of Users

The rate that users get is directly proportional to their distances from the eNB. The users will normally achieve minimal throughputs the further they get from their serving eNB, therefore, putting the cell edge users at the risk of very low QoS. When in collaboration however, as shown in Fig 4, the users are able to achieve a considerable high throughput. This feat however, depends on the marginal contribution of the user in order to uphold the principles of fairness.

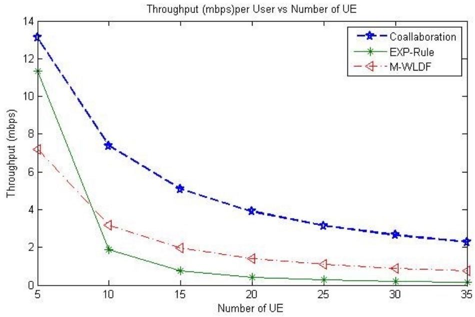

Fig 2: Throughput (mbps) per User vs Number of Users

For users at the cell edge to get higher throughput, there is need for more users in the neighbourhood to come together which is provisioned through the pathloss and required SINR. The more users in collaboration, the more payoff the coalition gets which has a direct impact on the throughput as in Fig 3.

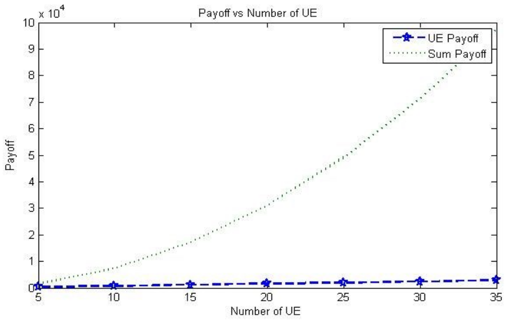

Fig 3: Payoff vs Number of UE

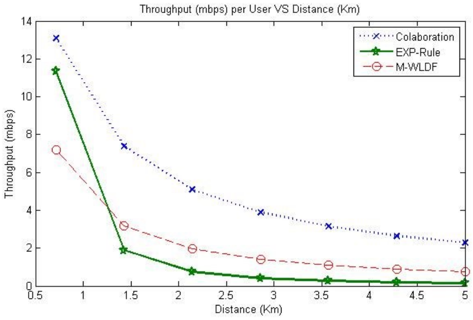

Fig 4: Throughput (mbps) per User vs Distance

VII. CONCLUSION

In this paper, we put our focus on the EXP-Rule and M-WLDF whose performance are compared with our algorithm. VoIP traffic was considered over a coverage area of 5km. Our algorithm proves to provide higher throughput for user than the EXP-Rule and M-WLDF. While they tend to achieve lower throughput when the number of users increases, our algorithm is rather resilient to the amount of load in the system and maintains a better performance than these two due to the fact that the number of participating users has a positive effect on the amount of payoff that users get which increases their throughput.

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