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ANALYTICAL SOLUTION OF THE MATHEMATICAL MODEL OF EBOLA DISEASE DYNAMICS INCORPORATING INFECTION-AGE STRUCTURE IN THE QUARANTINED COMPARTMENT WITH TREATMENT

****Azuaba Emmanuel and Akinwande Ninuola Ifeoluwa.***

Department of Mathematics, Federal University of Technology, Minna, Nigeria.

Abstract

In this work, we propose a mathematical model of Ebola Virus Disease (EVD) dynamics incorporating infection-age structure in the compartment of the quarantined with treatment. The model equations consist of Ordinary and Partial Differential Equations and Integro-Differential Equation. The analytical solutions of the model equations are obtained using the Differential Transformation Method (DTM). The Maple software package is used to present the graphical profiles of the solutions.

Keywords: Infection-Age, Ebola Virus Disease, Differential Transform Method

1. Introduction

Ebola Virus Disease (EVD), also known as Ebola haemorrhagic fever is a rare disease caused by Ebola virus strains. Ebola can cause disease in humans and nonhuman primates. According to World Health Organisation [1], Ebola virus is one of the deadliest diseases caused by infection from the family of RNA (Ribonucleic Acid) virus called *Filovirus.* There are five identified Ebola virus strains. Four of the five virus strains have caused disease in the humans: Ebola virus *(Zaire ebolavirus);* Sudan virus *(Sudan ebolavirus);* Tai Forest virus *(Tai Forest ebolavirus*formerly known as Cote d'Ivoire *ebolavirus);* and Bundibugyo virus *(bundibugyoebolavirus).* The fifth, Reston virus *(Reatonebolavirus)*, has caused disease in nonhuman primates such as monkeys, gorillas, fruit bats, forest antelope porcupines, and chimpanzees. In this paper, only the Zaire Ebola virus strain causing the actual outbreaks in West Africa that is considered. According to [2] ascertained that Zaire Ebola virus was first discovered in 1976 near the Ebola River in what is now the Democratic Republic of the Congo. Since then, outbreaks of Ebola virus disease appeared sporadically in Africa. In March 2014, the Ebola virus was first reported in Guinea, now the virus has spread to other West African nations like Liberia, Sierra Leone, Senegal and Nigeria. Ebola signs and symptoms of infection typically start between two days and three weeks after contracting the virus with a fever, sore throat, muscle pain, and headache. Then, vomiting, diarrhea and rash usually follow along with decreased function of the liver and kidneys. At this time some people begin to bleed both internally and externally [3]. The disease has a high risk of death, killing between 25 and 90 percent of those infected with an average of about 50 percent. This is often due to low blood pressure from fluid loss.

2. Material and Methods

2.1 Model Formulation

The model equations are formulated using ordinary and partial differential equations and integro-differential equation. The model flow diagram is shown in figure 2.1. The total population is partitioned into five (5) compartments namely: Susceptible $S(t)$; Quarantine $P(t)$ under observation; Quarantine $Q(t)$ with treatment; Infected $I(t)$; fully recovered $R(t)$ individuals due to permanent immunity and Dead individuals $D(t)$ that are generated from $I(t)$ and $Q(t)$ only and is not a compartment among the living but dead individuals that are capable of causing subsequent reinfection if not properly handle. Therefore, it is not included in the total population. The $S(t)$ is a class in which members are EVD free but are open to infection through contacts with $I(t)$ and $D(t)$ respectively at the rate α_1 and $\alpha_2(1-\theta)$ where α_1 is the effective contact rate

Corresponding Author: Azuaba E., Email: mlahaga35@gmail.com, Tel: +2348135819787

between $S(t)$ and $I(t)$, $\alpha_2(1-\theta)$ is the effective contact rate between $S(t)$ and $D(t)$, $(1-\theta)$ is the proportion of those who are not given proper burial and can cause subsequence reinfection, α_2 is the effective contact rate between $S(t)$ and $(1-\theta)$, hence $0 \le \theta \le 1$. $S(t)$ is generated through a natural birth rate Λ and they are reduced by a natural death rate μ. When $S(t)$ comes in contact with Ebola dead individuals $D(t)$ can become contaminated with the dead at $(1-\theta)$, all the people who are suspected of having been infected usually by contacts tracing of the infected are then move into a class known as Quarantine $P(t)$ under observation.

After observation by healthcare workers if they are not infectious, meaning they are Ebola free then they can be removed back to join $S(t)$ population at the rate ϕ where ϕ is a fraction of individuals who are not infected with EVD and is confirmed after observation. While individuals who are infectious progress into $Q(t)$ at the rate $(1-\phi)$. Where, $(1-\phi)$ is the probability of a newly observed individual to become infected and hence $0 \le \phi \le 1$. $P(t)$ is reduced at a natural death rate μ. $I(t)$ is a class of the individuals who are infected with EVD. This class is unavoidably present since Ebola infection is a severe disease and can cause death between two (2) to twenty one (21) days. Some of the infected individuals from $S(t)$ who are not suspected of having infection but suddenly become infected with EVD, they are automatically members of the infected class $I(t)$ and they are reduced due to μ, φ and δ_1. Where μ is the natural death rate, φ is the rate of quarantining $I(t)$ to $Q(t)$ and δ_1 is the disease induced death rate for $I(t)$. $D(t)$ is the compartment of dead individuals for both natural and Ebola induced death which are generated from $I(t)$ and $Q(t)$ through $\mu+\delta_1$ and $\mu+\delta_2$ respectively, where δ_1 and δ_2 are the disease induced death rates of $I(t)$ and $Q(t)$ respectively. $D(t)$ is usually reduced by proper burial at the rate θ whereby θ is a fraction which is devoid of subsequent reinfection. This class exists because they are capable of spreading EVD through contaminated burial practices at the rate of $\alpha_2(1-\theta)$. this is possible because of factors such as political, economic situations, cultural, wars and lack of appropriate Personal Protection Equipment (PPE). To prevent the spread of Ebola virus disease, confirmed cases of individuals who are infectious of Ebola virus disease are isolated to remain in the Quarantine with treatment class $Q(t)$. $Q(t)$ is the class of individuals that are quarantined (strict isolation zone) and are receiving treatment. This is the second quarantine class. In this class, individuals are generated from $P(t)$ and $I(t)$ through $(1-\phi)$ and φ respectively. They are reduced by ω and $\mu+\delta_2$ where ω is the treatment rate and $\mu+\delta_2$ is natural and disease induced death rate of $Q(t)$. $R(t)$ is generated from $Q(t)$ via a treatment rate ω and is reduced by a natural death rate μ. The Quarantine with treatment class $Q(t)$ is structured by the infection-age with the density function $q(t,\tau)$ following the idea by [4] where t is the time parameter and τ is the infection-age. There is a maximum infection-age T at which a member of the infected individuals from quarantine with treatment class $Q(t)$ must leave the compartment via death; and so $0 \le \tau \le T$. The death rate due to EVD infection is given by $\sigma(\tau) = \delta e^{-k(T-\tau)}$ where δ and k are constants. δ is the maximum death rate from infection while k is a control parameter which could be a measure of slowing down the death of the infected, this can be the measure of effectiveness of slowing down the death of the infected for example the effectiveness of ω. A high value of k will imply high effectiveness of such control measures and vice versa.

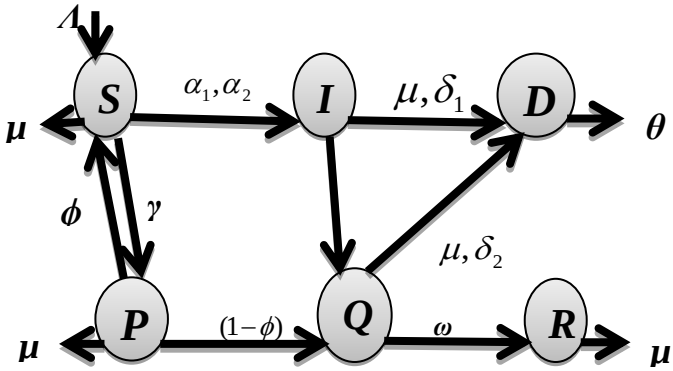

Figure 1: Model Diagram

Table 1: Notation and definition of variables and parameters in Model Diagram are represented as follows

Symbol	Description
$S(t)$	Susceptible individuals at time t
$P(t)$	Quarantine under observation individuals at time t
$I(t)$	Infected individuals at time t
$D(t)$	Dead individuals resulting from $I(t)$ and $Q(t)$ at time t
$R(t)$	Recovered individuals due to permanent recovery from infection at time t
$Q(t)$	Quarantine individuals with treatment at time t
Λ	Birth rate into $S(t)$ only
μ	Natural death rate
δ_1	Disease induced death rate of infected class $I(t)$
δ_2	Disease induced death rate of quarantine with treatment $Q(t)$
α_1	Effective contact rate between $I(t)$ and $S(t)$
α_2	Effective contact rate between $D(t)$ and $S(t)$
γ	Quarantine rate of suspected individuals from $S(t)$ to $P(t)$
φ	Quarantine rate from $I(t)$ to $Q(t)$
ϕ	Proportion of individuals that move back from $P(t)$ to $S(t)$ after observation
θ	Proportion of those who are given proper burial, $0 \le \theta \le 1$
ω	Treatment rate
$\sigma(\tau)$	Death rate from infection
δ	Maximum death rate from infection
k	Effectiveness efforts of slowing down the death of infected members
τ	Infection-age
T	Maximum age of infection

2.2 Model Equations

$$\frac{dS}{dt} = \Lambda - \frac{\alpha_1 IS}{N} - \frac{\alpha_2(1-\theta)DS}{N} + \phi P - (\mu + \gamma)S \tag{2.1}$$

$$\frac{dP}{dt} = \gamma S - (\phi + \mu)P \tag{2.2}$$

$$\frac{dI}{dt} = \frac{\alpha_1 IS}{N} + \frac{\alpha_2(1-\theta)DS}{N} - (\mu + \delta_1 + \varphi)I \tag{2.3}$$

$$\frac{dD}{dt} = (\mu + \delta_1)I + (\mu + \delta_2)Q - \theta D \tag{2.4}$$

$$\frac{dR}{dt} = \omega Q - \mu R \tag{2.5}$$

$$\frac{\partial q(t,\tau)}{\partial t} + \frac{\partial q(t,\tau)}{\partial \tau} + (\mu + \sigma(\tau))q(t,\tau) = 0 \tag{2.6}$$

The boundary conditions:

$$q(t,0) = B(t) = \frac{\alpha_1 IS}{N} + \frac{\alpha_2(1-\theta)DS}{N} + \varphi I + (1-\phi)P \tag{2.7}$$

$$q(t,T) = 0 \tag{2.8}$$

Where, T is the maximum infection age; i.e. when $\tau = T$ the infected member dies of EVD.

The total population size N is given by: $N(t) = S(t) + P(t) + I(t) + R(t) + Q(t)$ (2.9)

The total population of infected individuals from quarantine with treatment is given as:

$$Q(t) = \int_0^T q(t,\tau)d\tau \tag{2.10}$$

According to [5] the death rate via infection is given by (2.11)

$$\sigma(\tau) = \delta e^{-k(T-\tau)} \tag{2.11}$$

The initial conditions are:

$$S(0) = S_0,\ P(0) = P_0,\ I(0) = I_0,\ D(0) = I_0,\ R(0) = R_0,\ Q(0) = Q_0,\ N(0) = N_0 \tag{2.12}$$

$$q(0,\tau) = \psi(\tau) \tag{2.13}$$

Integrating (2.6) over τ gives:

$$\frac{dQ}{dt} + q(t,T) - q(t,0) = -\mu\int_0^T q(t,\tau)d\tau - \int_0^T \sigma(\tau)q(t,\tau)d\tau \tag{2.14}$$

Let,

$$\int_0^T \sigma(\tau)q(t,\tau)d\tau = \delta\int_0^T e^{-k(T-\tau)}q(t,\tau)d\tau \tag{2.15}$$

Where, $\sigma(\tau)$ and $\int_0^T q(t,\tau)d\tau$ are as defined in (2.11) and (2.10) respectively.

Applying integration by parts on R. H.S. of (2.15) we get:

$$\int_0^T \sigma(\tau)q(t,\tau)d\tau = 0 \tag{2.16}$$

Substituting equations (2.7), (2.8), (2.10) and (2.16) into (2.14) yields

$$\frac{dQ}{dt} = \frac{\alpha_1 IS}{N} + \frac{\alpha_2(1-\theta)DS}{N} + \varphi I + (1-\phi)P - \mu Q \tag{2.17}$$

2.3 Analytical Solution of the Model Equations using Differential Transformation Method (DTM)

DTM is a semi analytical-numerical technique that depends on Taylor Series. It was first introduced by Zhou in a study about electrical circuits [6]. It is possible to solve differential equations, difference equations, differential difference equations, fractional differential equations, pantograph equations and integro-differential equations by using this method [7, 8]. In this section we apply Differential Transformation Method (DTM) to the model equations. We set the model equations to be a function of $q_1(t)$, $q_1(t)$ can be expanded in Taylor series about a point $t = 0$ as:

$$q_1(t) = \sum_{k=0}^{\infty} \frac{t^k}{k!}\left[\frac{d^k q_1}{dt^k}\right]_{t=0} \tag{2.18}$$

Where,

$$q_1(t) = s(t) = p(t) = i(t) = d(t) = r(t) \tag{2.19}$$

The differential transformation of $q_1(t)$ is defined as:

$$Q_1(t) = \frac{1}{k!}\left[\frac{d^k q_1}{dt^k}\right]_{t=0} \tag{2.20}$$

Where,

$$Q_1(t) = S(t) = P(t) = I(t) = D(t) = R(t) \tag{2.21}$$

and the inverse differential transformation is defined by:

$$q_1(t) = \sum_{k=0}^{\infty} t^k Q_1(t) \tag{2.22}$$

Table 2: The following table is the Fundamental Mathematical Operations by Differential Transformation Method (DTM)

Original Function	**Transformation Function**
$f(x) = g(x)h(x)$	$F(k) = \sum_{m=0}^{k} G(m)H(k-m).$
$f(x) = g(x)\int_{x_0}^{x} h(t)dt$	$F(k) = \sum_{m=1}^{k} \frac{1}{m} G(k-m)H(m-1),$ $k \geq 1.$
$f(x) = cg(x)$	$F(k) = cG(k),$ Where c is a constant.

Sources: [9, 10].

We note that, the transformations of integrals are considered for $k \geq 1$according to the third row in table 2. Before the application of DTM on equations (2.1)-(2.5), equation (2.6) was solved using the characteristics method and we obtained:

$$q(t,\tau) = \psi(\tau)g(t) \quad (2.23)$$

Where,

$$g(t) = e^{-(\mu+\sigma(\tau))t} \quad (2.24)$$

2.4 Numerical Simulation and Method

In this section, we shall consider Sierra Leone for the illustration. Sierra Leone is one of the three geographically endemic areas of EVD in West Africa. The variables and values in table 3,are the 2014 to 2016 data of EVD outbreak in Sierra Leone [11], [12] and [13].

Table3: Variablesandvalues of EVD outbreak in Sierra Leone:

Year	**P(t)**	**I(t)**	**Q(t)**	**D(t)**	**R(t)**
2014	1805	9446	7354	2758	6619
2015	5131	14122	8704	3955	7660
2016	5131	14124	8706	3956	6965
Total	12067	37692	24764	10669	21244

Table 4:Initial parameters and estimated values used for Computer simulation

Parameter	**Value**
α_1	0.00000197
α_2	0.00000148
Λ	10970
θ	0.85
ϕ	0.55
μ	0.0017
φ	0.75
δ_1	0.0212
δ_2	0.0036
$\sigma(0) = \delta$	0.0248
$\psi(0)$	24764
ω	0.75
γ	1.429
k	0

According to [14], the total population of Sierra Leone was estimated at 6453000. Therefore, the total number of susceptible individuals in the Sierra Leone is obtained as follows:

$$S = N - (I + P + Q + D + R) \quad (2.25)$$

Equation (2.25) yields:

$$S = 6453000 - (37692 + 12067 + 24764 + 10669 + 21244) = 6346564 \quad (2.26)$$

From table 2.4 and equation (2.11), we obtain:

$$\sigma(\tau) = \delta \quad (2.27)$$

Substituting (2.27) in (2.24) yields:

$$g(t) = e^{-(\mu+\delta)t} \quad (2.28)$$

From table 2.4 and (2.28) we obtain the followings:

$$g(0) = 1,\ g(1) = 0.9738,\ g(2) = 0.9484,\ for\ t = 0, 1, 2, \ldots \quad (2.29)$$

So that:

$$g(0) = G(0),\ g(1) = G(1),\ g(2) = G(2),\ \ldots \quad (2.30)$$

From equations (2.29) and (2.30), this implies that:

$$G(0)=1, G(1)=0.9738, G(2)=0.9484, \ldots \quad (2.31)$$

We observed that if the infection age has reached its maximum point, i.e. $\tau = T$ an individual will die of the disease no matter how the control measures are high. This agrees with our intuitive reasoning and hence we deduced from the foregoing that:

$\psi(0)=24764$ for all values of $\tau = 0, 1, 2, 3, \ldots$

Since $0 \le \tau \le T$, following from the above, we have that:

$$\psi(0) \approx \psi(1) \approx \psi(2) \quad (2.32)$$

Therefore, equations (2.1)-(2.5) are re-written as follows:

$$\frac{dS}{dt} = \Lambda - \frac{\alpha_1 SI}{N} - \frac{\alpha_2(1-\theta)SD}{N} + \phi P - (\mu+\gamma)S \quad (2.33)$$

$$\frac{dP}{dt} = \gamma S - (\mu+\phi)P \quad (2.34)$$

$$\frac{dI}{dt} = \frac{\alpha_1 SI}{N} + \frac{\alpha_2(1-\theta)SD}{N} - (\mu+\delta_1+\varphi)I \quad (2.35)$$

$$\frac{dD}{dt} = (\mu+\delta_2)g(t)\int_0^T \psi(\tau)d\tau + (\mu+\delta_1)I - \theta D \quad (2.36)$$

$$\frac{dR}{dt} = \omega g(t)\int_0^T \psi(\tau)d\tau - \mu R \quad (2.37)$$

Where,

$$g(t)\int_0^T \psi(\tau)d\tau = Q \quad (2.38)$$

Equation (2.38) was obtained from (2.10) & (2.23). Therefore, from equations (2.33) to (2.37) we solved by utilising the fundamental operations of DTM in table 2, we obtain the following recurrence relation of equations as follows:

$$S(k+1) = \frac{1}{k+1}\left[\begin{array}{l}\Lambda - \frac{\alpha_1}{N}\sum_{m=0}^{k} S(m)I(k-m) - \frac{\alpha_2(1-\theta)}{N}\sum_{m=0}^{k} S(m)D(k-m) \\ +\phi P(k) - (\mu+\gamma)S(k)\end{array}\right] \quad (2.39)$$

$$P(k+1) = \frac{1}{k+1}\left[\gamma S(k) - (\mu+\phi)P(k)\right] \quad (2.40)$$

$$I(k+1) = \frac{1}{k+1}\left[\begin{array}{l}\frac{\alpha_1}{N}\sum_{m=0}^{k} S(m)I(k-m) + \frac{\alpha_2(1-\theta)}{N}\sum_{m=0}^{k} S(m)D(k-m) \\ -(\mu+\delta_1+\varphi)I(k)\end{array}\right] \quad (2.41)$$

$$D(k+1) = \frac{1}{k+1}\left[(\mu+\delta_2)\sum_{m=1}^{k} G(k-m)\psi(m-1) + (\mu+\delta_1)I(k) - \theta D(k)\right] \quad (2.42)$$

$$R(k+1) = \frac{1}{k+1}\left[\omega\sum_{m=1}^{k} G(k-m)\psi(m-1) - \mu R(k)\right] \quad (2.43)$$

Where,

$k = 0, 1, 2, 3, \ldots$

Substituting $k=0$ into (2.39) to (2.43) gives:

$$S(1) = \left[\Lambda - \frac{\alpha_1}{N}S(0)I(0) - \frac{\alpha_2(1-\theta)}{N}S(0)D(0) + \phi P(0) - (\mu+\gamma)S(0)\right] \quad (2.44)$$

$$P(1) = \left[\gamma S(0) - (\mu+\phi)P(0)\right] \quad (2.45)$$

$$I(1) = \left[\frac{\alpha_1}{N}S(0)I(0) + \frac{\alpha_2(1-\theta)}{N}S(0)D(0) - (\mu+\delta_1+\varphi)I(0)\right] \quad (2.46)$$

$$D(1) = \left[(\mu+\delta_1)I(0) - \theta D(0)\right] \quad (2.47)$$

$$R(1) = \left[-\mu R(0)\right] \quad (2.48)$$

Substituting $k=1$ into (2.39) to (2.43) gives:

$$S(2)=\frac{1}{2}\left[\begin{array}{l}\Lambda-\frac{\alpha_1}{N}(S(0)I(1)+S(1)I(0))-\frac{\alpha_2(1-\theta)}{N}(S(0)D(1)+S(1)D(0))\\+\phi P(1)-(\mu+\gamma)S(1)\end{array}\right] \quad (2.49)$$

$$P(2)=\frac{1}{2}\left[\gamma S(1)-(\mu+\phi)P(1)\right] \quad (2.50)$$

$$I(2)=\frac{1}{2}\left[\frac{\alpha_1}{N}(S(0)I(1)+S(1)I(0))+\frac{\alpha_2(1-\theta)}{N}(S(0)D(1)+S(1)D(0))-(\mu+\delta_1+\varphi)I(1)\right] \quad (2.51)$$

$$D(2)=\frac{1}{2}\left[(\mu+\delta_2)G(0)\psi(0)+(\mu+\delta_1)I(1)-\theta D(1)\right] \quad (2.52)$$

$$R(2)=\frac{1}{2}\left[\omega G(0)\psi(0)-\mu R(1)\right] \quad (2.53)$$

Substituting $k=2$ into (2.39) to (2.43)gives:

$$S(3)=\frac{1}{3}\left[\begin{array}{l}\Lambda-\frac{\alpha_1}{N}(S(0)I(2)+S(1)I(1)+S(2)I(0))\\-\frac{\alpha_2(1-\theta)}{N}(S(0)D(2)+S(1)D(1)+S(2)D(0))+\phi P(2)-(\mu+\gamma)S(2)\end{array}\right] \quad (2.54)$$

$$P(3)=\frac{1}{3}\left[\gamma S(2)-(\mu+\phi)P(2)\right] \quad (2.55)$$

$$I(3)=\frac{1}{3}\left[\begin{array}{l}\frac{\alpha_1}{N}(S(0)I(2)+S(1)I(1)+S(2)I(0))\\+\frac{\alpha_2(1-\theta)}{N}(S(0)D(2)+S(1)D(1)+S(2)D(0))-(\mu+\delta_1+\varphi)I(2)\end{array}\right] \quad (2.56)$$

$$D(3)=\frac{1}{3}\left[(\mu+\delta_2)\left(G(1)\psi(0)+\frac{1}{2}G(0)\psi(1)\right)+(\mu+\delta_1)I(2)-\theta D(2)\right] \quad (2.57)$$

$$R(3)=\frac{1}{3}\left[\omega\left(G(1)\psi 0)+\frac{1}{2}G(0)\psi(1)\right)-\mu R(2)\right] \quad (2.58)$$

Substituting $k=3$ into (2.39) to (2.43) gives:

$$S(4)=\frac{1}{4}\left[\begin{array}{l}\Lambda-\frac{\alpha_1}{N}(S(0)I(3)+S(1)I(2)+S(2)I(1)+S(3)I(0))\\-\frac{\alpha_2(1-\theta)}{N}(S(0)D(3)+S(1)D(2)+S(2)D(1)+S(3)D(0))+\phi P(3)-(\mu+\gamma)S(3)\end{array}\right] \quad (2.59)$$

$$P(4)=\frac{1}{4}\left[\gamma S(3)-(\mu+\phi)P(3)\right] \quad (2.60)$$

$$I(4)=\frac{1}{4}\left[\begin{array}{l}\frac{\alpha_1}{N}(S(0)I(3)+S(1)I(2)+S(2)I(1)+S(3)I(0))\\+\frac{\alpha_2(1-\theta)}{N}(S(0)D(3)+S(1)D(2)+S(2)D(1)+S(3)D(0))-(\mu+\delta_1+\varphi)I(3)\end{array}\right] \quad (2.61)$$

$$D(4)=\frac{1}{4}\left[(\mu+\delta_2)\left(G(2)\psi(0)+\frac{1}{2}G(1)\psi(1)+\frac{1}{3}G(0)\psi(2)\right)+(\mu+\delta_1)I(3)-\theta D(3)\right] \quad (2.62)$$

$$R(4)=\frac{1}{4}\left[\omega\left(G(2)\psi(0)+\frac{1}{2}G_2(1)\psi(1)+\frac{1}{3}G(0)\psi(2)\right)-\mu R(3)\right] \quad (2.63)$$

And from (2.38), we utilised the definition of transform as shown in the 3rd row of table (2) and yields:

$$Q(k)=\sum_{m=1}^{k}\frac{1}{m}G(k-m)\psi(m-1),k\geq 1 \quad (2.64)$$

Where,

$k=1, 2, 3, \ldots$

Substituting $k=1$ into (2.64) gives:

$$Q(1)=G(0)\psi(0) \quad (2.65)$$

Substituting $k=2$ into (2.64) gives:

$$Q(2)=G(1)\psi(0)+1/2G(0)\psi(1) \quad (2.66)$$

Substituting $k=3$ into (2.64) gives:

$$Q(3)=G(2)\psi(0)+\frac{1}{2}G(1)\psi(1)+\frac{1}{3}G(0)\psi(2) \quad (2.67)$$

Substituting the values in table 3 and 4, equations (2.44) to (2.63) into (2.22) gives:

$$S(t)=\sum_{k=0}^{\infty}S(k)t^k = 6346564+9076058t-3994863t^2+2639377.13t^3-361271.27t^4+\ldots$$

$$P(t)=\sum_{k=0}^{\infty}P(k)t^k = 12067+9062582.6t+3984929.73t^2-2635714.79t^3+1306448.44t^4+\ldots$$

$$I(t)=\sum_{k=0}^{\infty}I(k)t^k = 37692-29132.1t+11258.11t^2-2900.50t^3+543.89t^4+\ldots$$

$$D(t)=\sum_{k=0}^{\infty}D(k)t^k = 10669-8205.5t+3219.40t^2-761.75t^3+203.30t^4+\ldots$$

$$R(t)=\sum_{k=0}^{\infty}R(k)t^k = 21244-36.11t+9286.53t^2+9119.03t^3+8208.33t^4+\ldots \quad (2.68)$$

Similary; from table 4, euquations (2.31) to (2.32) and equations (2.65) to (2.67) we obtain:

$$Q(t)=\sum_{k=1}^{\infty}Q(k)t^k = 24764t+36497.18t^2+43798.44t^3+\ldots \quad (2.69)$$

Therefor, equations (2.68) and (2.69) is the solution of the model equations. The computation of equations (2.68) and (2.69) was done using Microsoft Excel (Spreadsheet) and hence parameter values can be varied.

3. Results and Discussion

In this section, we consider the graphical solution of the DTM solution. A Maple 17 software package is used to present the graphical profiles of the solution.

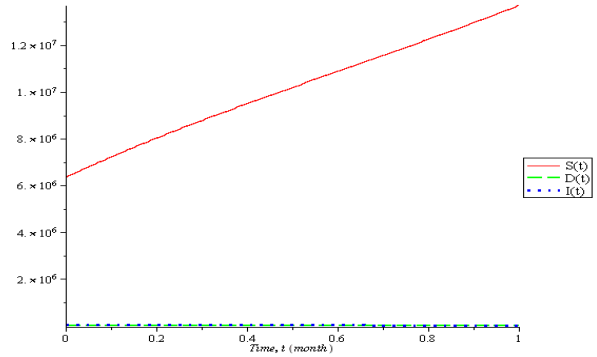

Figure 2: Effect of High Proper Burial Rate on ***S(t), I(t)*** and ***D(t).***

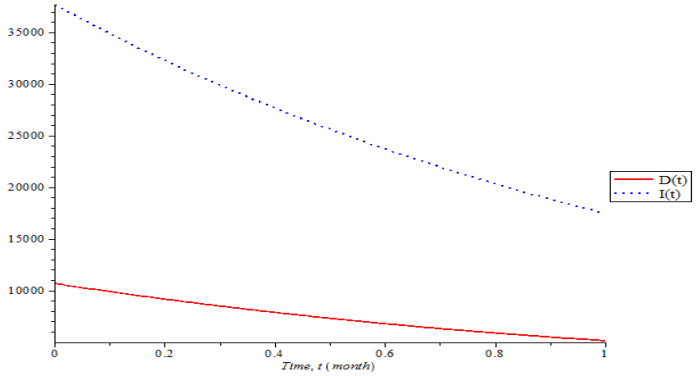

Figure 3: Effect of High Proper Burial Rate on ***I(t)*** and ***D(t).***

Figure 2 shows the effect of high proper burial rate on the susceptible, infected and dead individuals respectively. It is shown that the susceptible population is increased and the infected population decreased to zero. This shows that high proper burial of Ebola deceased is a good control measure.

It is shown from figure 3, that high proper burial rate of Ebola deceased also decreased the infected population.

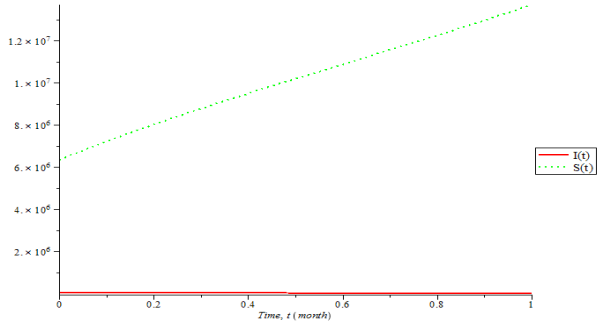

Figure 4: Effect of High Proper Burial rate on ***I(t)*** and ***S(t).***

High proper burial rate has much effect on the Ebola disease dynamics as seen in figure 4. At first the infected population rose a little but swiftly turns to zero and the susceptible population shows steady increased.

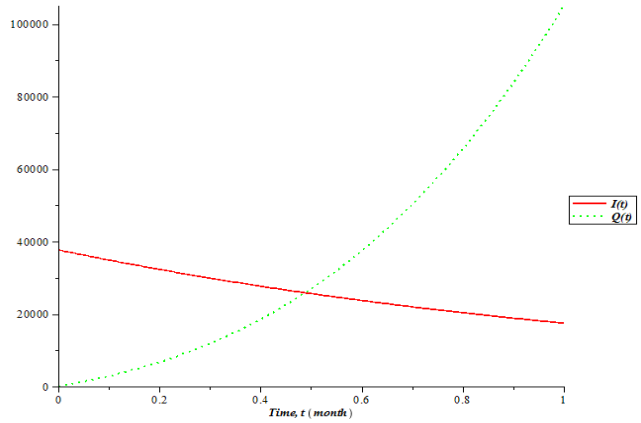

Figure 5: Effect of High Proper Burial Rate on ***I(t)*** and ***Q(t).***

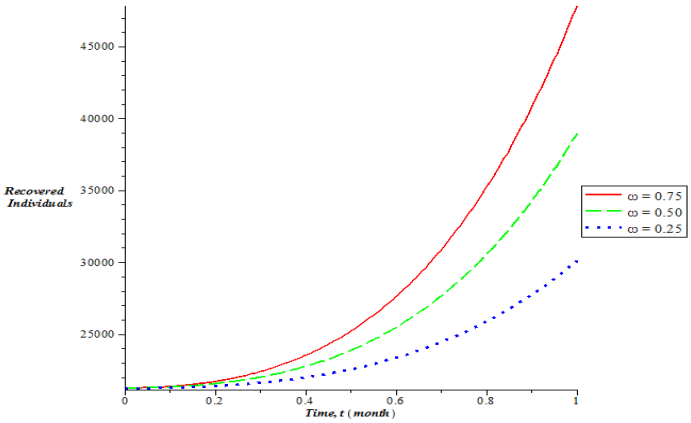

Figure 6: Effect of Different Treatment Rate treatment on ***R(t).***

It is observed from figure 5 that infected population decreased due to high proper burial of Ebola deceased and the quarantine with treatment population increased. This shows that quarantine with treatment is an effective control measure.

Figure 6, shows the effect of different treatment rate on recovered population. The recovered population increased with high treatment rate and decreased with low treatment rate. This implies that high medical supportive care could be a better control strategy.

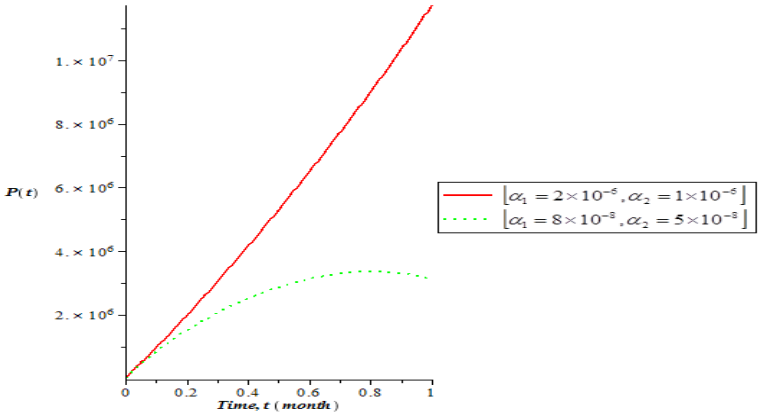

Figure 7: Effect of Different Rate of α_1 and α_2 on ***P (t)*** respectively.

Figure 7 shows the effect of two different infection rates on quarantine under observation; effective contact rate between the infected and susceptible population, and effective contact rate between the deceased and susceptible population. It is observed that the quarantine under observation increased with different high rate of the two infection rates and started decreasing simultaneously with different low rate of the two infection rates.

4. Conclusion

In this work, it is observed that the DTM is robust but applicable to ordinary and partial differential equations and integro-differential equation. The method gives rapidly converging series solutions. The series solutions obtained with DTM can be written in exactly closed form. Furthermore, Ebola virus by nature is very infectious and is fatal since there is no proper medical treatment as of moment. This in effect showed that the rate of contact per infective in unit time is constant. As shown by our mathematical model, an uncontrolled transmittable contacts between the infected and Ebola deceased to the susceptible is capable of causing high mortality rate. This is evidence in the last Ebola outbreaks of 2014 to 2016 in West Africa most especially that of Sierra Leone and recorded the worst case scenario such as highest dead rate, much more than the total of all death since the discovery of the virus in 1976. However, a better management option as revealed by the model is the proper burial of Ebola deceased and aggressive quarantine system with infection-age structured to enhance proper medical care and support which ultimately reduced the rate of infection by our analytical solutions. Since by nature quarantine restrict the movement of infective individuals, government should put more effort in high quarantine strategies whenever there is Ebola outbreak and people should be sensitised and given personal protection equipment (PPE) when handling Ebola deceased victims in order to avoid subsequent reinfection.

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